Sample Question Paper - 18
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed: 120 minutes **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

SECTION A

1. Find the value of k for which the roots of the quadratic equation $2x^2 + kx + 8 = 0$ will have the equal roots ?

OR

Find the roots of the equation $x^2 + 7x + 10 = 0$

2. Which term of the AP $3, 12, 21, 30, ...$ will be 90 more than its 50^{th} term.

3. Prove that the lengths of two tangents drawn from an external point to a circle are equal.

4. Find the number of plates, 1.5 cm in diameter and 0.2 cm thick, that can be fitted completely inside a right circular of height 10 cm and diameter 4.5 cm.

5. Write the median class of the following distribution :

Classes	0- 10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	4	8	10	12	8	4

6. The following are the ages of 300 patients getting medical treatment in a hospital on a particular day :

Age (in years)	10-20	20-30	30-40	40-50	50-60	60-70
Number of students	60	42	55	70	53	20

Form the "less than type" cumulative frequency distribution table.

OR

Find the mean of the data using an empirical formula when it is given that mode is 50.5 and median in 45.5.

Section B

7. Solve the following quadratic equation for x :

$9x^2 - 9(a+b)x + 2a^2 + 5ab + 2b^2 = 0$

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8. The 14^{th} term of an AP is twice its 8^{th} term. If the 6^{th} term is -8, then find the sum of its first 20 terms.

9. The person standing on the bank of river observes that the angle of elevation of the top of a tree standing on opposite bank is $60°$. When he moves 30 m away from the bank, he finds the angle of elevation to be $30°$. Find the height of tree and width of the river.

10. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.

OR

Draw a circle of radius 3.5 cm. From a point P, 6 cm from its centre, draw two tangents to the circle.

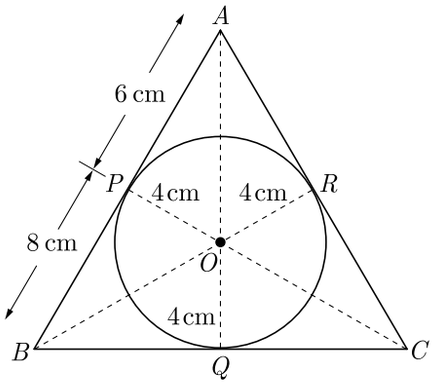
Section C

11. From the top of a tower of height 50 m, the angles of depression of the top and bottom of a pole are $30°$ and $45°$ respectively. Find :

(i) How far the pole is from the bottom of the tower,

(ii) The height of the pole. (Use $\sqrt{3} = 1.732$)

12. In Figure the radius of incircle of ΔABC of area $84\,cm^2$ and the lengths of the segments AP and BP into which side AB is divided by the point of contact are 6 cm and 8 cm Find the lengths of the sides AC and BC.



OR

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

13. The advantages of cone bottom tanks are found in nearly every industry, especially where getting every last drop from the tank is important. This type of tank has excellent geometry for draining, especially with high solids content slurries as these cone tanks provide a better full-drain solution. The conical tank eliminates many of the problems that flat base tanks have as the base of the tank is sloped towards the centre giving the greatest possible

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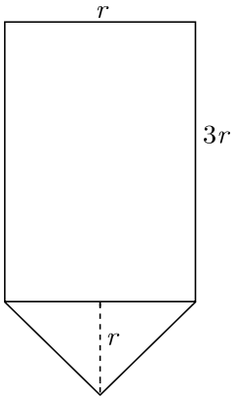
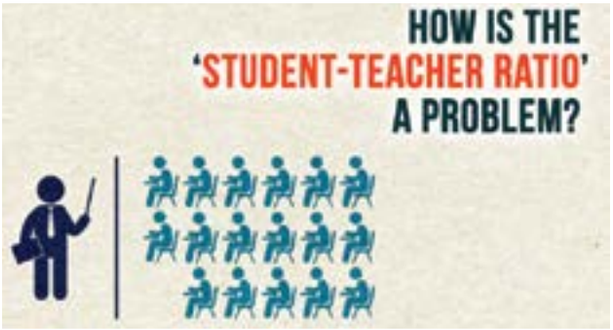
full-drain system in vertical tank design.

Rajesh has been given the task of designing a conical bottom tank for his client. Height of conical part is equal to its radius. Length of cylindrical part is the 3 times of its radius. Tank is closed from top. The cross section of conical tank is given below.



(i) If radius of cylindrical part is taken as 3 meter, what is the volume of above conical tank ?
(ii) What is the area of metal sheet used to make this conical tank ? Assume that tank is covered from top.

14. Student-teacher ratio expresses the relationship between the number of students enrolled in a school and the number teachers employed by the school. Student-teacher ratio is important for a number of reasons. It can be used as a tool to measure teacher workload as well as the allocation of resources. A low student-teacher ratio indicates the burden on a single teacher of teaching multiple students as well as the lack of time that each student gets.



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A survey was conducted in the 100 secondary school of Rajasthan and following frequency distribution table was prepared

Students per teacher	Number of School
20-25	5
25-30	15
30-35	25
35-40	30
40-45	15
45-50	10

(i) What is the median value of students per teacher?
(ii) What is the model value of students per teacher ?

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Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

Time Allowed: 120 minutes **Maximum Marks: 40**

General Instructions:

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4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

SECTION A

1. Find the value of k for which the roots of the quadratic equation $2x^2+kx+8=0$ will have the equal roots ?

Ans :

We have $2x^2+kx+8=0$

Comparing with $ax^2+bx+c=0$ we get

$a=2, b=k$, and $c=8$

For equal roots, $D=0$

$$\begin{aligned} b^2-4ac &= 0 \\ k^2-4\times 2\times 8 &= 0 \\ k^2 &= 64 \\ k &= \pm\sqrt{64} \end{aligned}$$

Thus $k=\pm 8$

OR

Find the roots of the equation $x^2+7x+10=0$

Ans :

We have,

$$\begin{aligned} x^2+7x+10 &= 0 \\ x^2+5x+2x+10 &= 0 \\ x(x+5)+2(x+5) &= 0 \\ (x+5)(x+2) &= 0 \\ x &= -5-2 \end{aligned}$$

So, roots of given equation are -5 and -2.

2. Which term of the AP $3, 12, 21, 30, \ldots..$ will be 90 more than its 50^{th} term.

Ans :

Let the first term be a, common difference be d and nth term be a_n.

We have $a=3, d=9$

Now $a_n = a+(n-1)d$

$a_{50} = 3+49\times 9 = 444$

Now, $a_n - a_{50} = 90$

$$\begin{aligned} 3+(n-1)9-444 &= 90 \\ (n-1)9 &= 90+441 \\ (n-1) &= \frac{531}{9}=49 \\ n &= 49+1=50 \end{aligned}$$

3. Prove that the lengths of two tangents drawn from an external point to a circle are equal.

Ans :

Consider a circle of radius r and centre at O as shown in figure below. Here we have drawn two tangent from P at A and B. We have to prove that

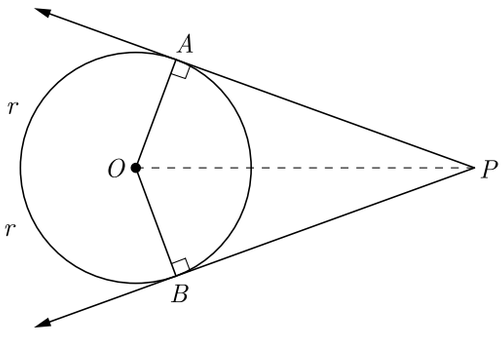
$AP = PB$

We join OA, OB and OP. In ΔPAO and ΔPBO, OP is common and $OA=OB$ radius of same circle. Since radius is always perpendicular to tangent, at point of contact,

$\angle OAP = \angle OBP = 90^\circ$

Thus $\Delta PAO \cong \Delta PBO.$

and hence, $AP = BP$



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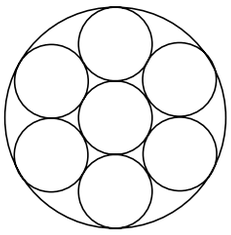
Thus length of 2 tangents drawn from an external point to a circle are equal.

4. Find the number of plates, 1.5 cm in diameter and 0.2 cm thick, that can be fitted completely inside a right circular of height 10 cm and diameter 4.5 cm.

Ans :

As per question we can arrange circular plate in right circular as follows. Here smaller circle is plate of 1.5 cm diameter and large circle is cylinder of 4.5 cm diameter.



From figure it may be easily seen that 6 plate will be fitted in cylinder in one layer.
Height of six plate is 0.2 cm. Total height of cylinder is 10 cm. Thus layer of plate in cylinder is $\frac{10}{0.2} = 50$ layer. Thus total plate $50 \times 6 = 300$.

5. Write the median class of the following distribution :

Classes	0 - 10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	4	8	10	12	8	4

Ans :

We prepare following cumulative frequency table to find median class.

Classes	Frequency	Less than c.f.
0-10	4	4
10-20	4	8
20-30	8	16
30-40	10	26
40-50	12	38
50-60	8	46
60-70	4	50
	$N = 50$	

We have $N = 50$; $\frac{N}{2} = 25$

Cumulative frequency just greater than $\frac{N}{2}$ is 26 and the corresponding class is 30-40. Thus median class is 20-20.

6. The following are the ages of 300 patients getting medical treatment in a hospital on a particular day :

Age (in years)	10-20	20-30	30-40	40-50	50-60	60-70
Number of students	60	42	55	70	53	20

Form the "less than type" cumulative frequency distribution table.

Ans :

Age	Number of Patients
Less then 20	60
Less then 30	102
Less then 40	157
Less then 50	227
Less then 60	280
Less then 70	300

OR

Find the mean of the data using an empirical formula when it is given that mode is 50.5 and median in 45.5.

Ans :

Mode, $M = 50.5$

Median, $M_d = 45.5$

Now $3M_d = M_o + 2M$

$3 \times 45.5 = 50.5 + 2M$

Mean, $M = \frac{136.5 - 50.5}{2} = 43$

Hence mean is 43.

Section B

7. Solve the following quadratic equation for x :

$9x^2 - 9(a+b)x + 2a^2 + 5ab + 2b^2 = 0$

Ans :

We have $9x^2 - 9(a+b)x + 2a^2 + 5ab + 2b^2 = 0$

Now $2a^2 + 5ab + 2b^2 = 2a^2 + 4ab + ab + 2b^2$

$= 2a[a+2b] + b[a+2b]$

$= (a+2b)(2a+b)$

Hence the equation becomes

$9x^2 - 9(a+b)x + (a+2b)(2a+b) = 0$

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$$9x^2 - 3[3a+3b]x + (a+2b)(2a+b) = 0$$

$$9x^2 - 3[(a+2b)+(2a+b)]x + (a+2b)(2a+b) = 0$$

$$9x^2 - 3(a+2b)x - 3(2a+b)x + (a+2b)(2a+b) = 0$$

$$3x[3x-(a+2b)] - (2a+b)[3x-(a+2b)] = 0$$

$$[3x-(a+2b)][3x-(2a+b)] = 0$$

$$3x-(2a+b) = 0$$

$$x = \frac{a+2b}{3}$$

$$3x-(2a+b) = 0$$

$$x = \frac{2a+b}{3}$$

Hence, roots are $\frac{a+2b}{3}$ and $\frac{2a+b}{3}$.

8. The 14^{th} term of an AP is twice its 8^{th} term. If the 6^{th} term is -8, then find the sum of its first 20 terms.

Ans :

Let the first term be a, common difference be d, n th term be a_n and sum of n term be S_n.
Here, $a_{14} = 2a_8$ and $a_6 = -8$

Now $a+13d = 2(a+7d)$

$$a+13d = 2a+14d$$

$$a = -d \quad ...(1)$$

and $a_6 = -8$

$$a+5d = -8 \quad ...(2)$$

Solving (1) and (2), we get

$$a = 2, d = -2$$

Now $S_{20} = \frac{20}{2}[2\times 2+(20-1)(-2)]$

$$= 10[4+19\times(-2)]$$

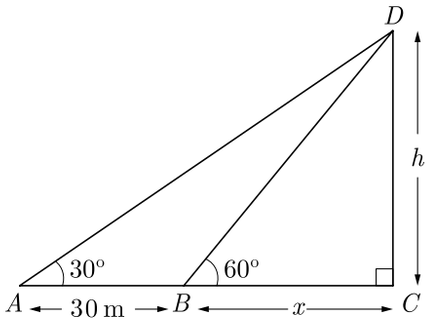
$$= 10(4-38)$$

$$= 10\times(-34) = -340$$

9. The person standing on the bank of river observes that the angle of elevation of the top of a tree standing on opposite bank is 60°. When he moves 30 m away from the bank, he finds the angle of elevation to be 30°. Find the height of tree and width of the river.

Ans :

Let CD be the tree of height h. Let A be the position of person after moving 30 m away from point B on bank of river. Let $BC = x$ be the width of the river.
As per given in question we have drawn figure below.



In right $\Delta DBC, \frac{h}{x} = \tan 60^\circ$

$$h = \sqrt{3}\,x \quad ...(1)$$

In right ΔADC,

$$\frac{h}{x+30} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}\,h = x+30 \quad ...(2)$$

Substituting the value of h from eq. (1) in eq. (2), we get

$$3x = x+30$$

$$x = 15 \text{ m} \quad ...(3)$$

Thus $h = \sqrt{3}\times 15 = 15\sqrt{3}$

$$= 15\times 1.732 = 25.98 \text{ m}$$

Hence, height of tree is 25.98 m and width of river is 15 m.

10. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.

Ans :

Steps of Construction :

1. Draw a line segment $AB = 8$ cm.
2. Draw a circle with centre A and radius 4 cm, draw another circle with centre B and radius 3 cm.
3. Bisect the line segment AB. Let its mid-point be M.
4. With centre as M and MA (or MB) as radius, draw a circle such that it intersects the two circles at points P, Q, R and S.
5. Join BP and BQ.
   Thus, BP and BQ are the required two tangents from B to the circle with centre A.

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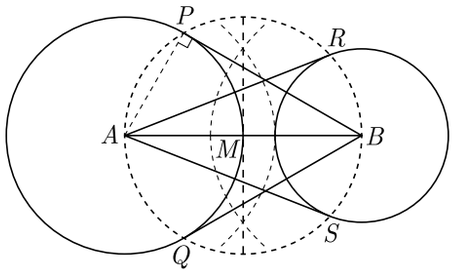
6. Join RA and SA.
Thus, RA and SA are the required two tangents from A to the circle with centre B.



Justification :
Let us join A and P.

$\angle APB = 90^\circ$ (Angle in a semi-circle)

$BP \perp AP$

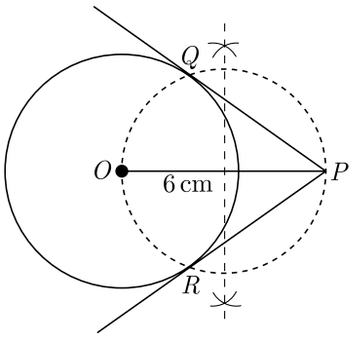
But AP is a radius of the circle with centre A. Thus BP has to be a tangent to the circle with centre A. Similarly, BQ has to be tangent to the circle with centre A.
Also, AR and AS are tangents to the circle with centre B.

OR

Draw a circle of radius 3.5 cm. From a point P, 6 cm from its centre, draw two tangents to the circle.

Ans :

Step of construction :
1. Draw a line segment OP of length 6 cm.
2. From the point O, draw a circle of radius $= 3.5$ cm.
3. Draw a perpendicular bisector of OP. Let M be the mid point of OP.
4. Taking M as centre and OM as radius draw a circle.
5. This circle intersects the given circle at Q and R.
6. Join PQ and PR, which are tangents to the circles.

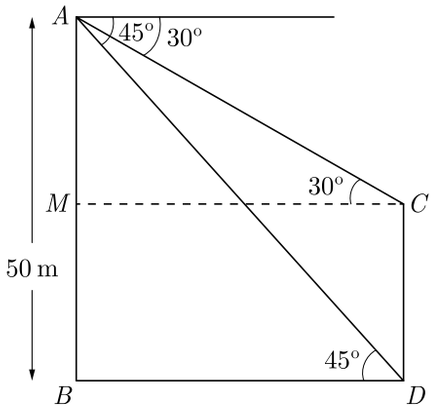


Section C

11. From the top of a tower of height 50 m, the angles of depression of the top and bottom of a pole are 30° and 45° respectively. Find :
(i) How far the pole is from the bottom of the tower,
(ii) The height of the pole. (Use $\sqrt{3} = 1.732$)

Ans :

Let AB be the tower of height 50 m and CD be the pole of height h. From the top of a tower of height 50 m, the angles of depression of the top and bottom of a pole are 30° and 45° respectively. As per given in question we have drawn figure below.



In right ΔABD we have,

$$\tan 45^\circ = \frac{AB}{BD} = 1$$

$$1 = \frac{50}{x} \Rightarrow x = 50 \text{ m}$$

(i) Thus distance of pole from bottom of tower is 50 m.

Now in ΔAMC we have

$$\tan 30^\circ = \frac{AM}{MC} = \frac{AM}{x}$$

$$AM = \frac{50}{\sqrt{3}} \text{ or } 28.87 \text{ m.}$$

(ii) Height pole $h = CD = BM$

$= 50 - 28.87 = 21.13$ m.

12. In Figure the radius of incircle of ΔABC of area $84\,\text{cm}^2$ and the lengths of the segments AP and BP into which side AB is divided by the point of contact are 6 cm and 8 cm Find the lengths of the

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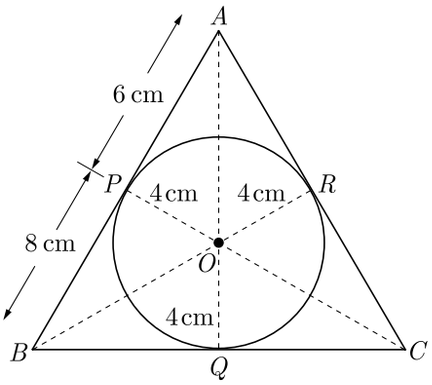
sides AC and BC.



Ans :

Since length of tangents from an external point to a circle are equal,

At A, $AP = AR = 6$ cm (1)

At B, $,BP = BQ = 8$ cm (2)

At C, $CR = CQ = x$ (3)

Perimeter of ΔABC,

$$p = AP + PB + BQ + QC + CR + RA$$
$$= 6 + 8 + 8 + x + x + 6 = 28 + 2x$$

Now area $\Delta ABC = \frac{1}{2}rp$

Here $r = 4$ is the radius of circle. Substituting all values we have

$$84 = \tfrac{1}{2} \times 4 \times (28 + 2x)$$
$$84 = 56 + 4x$$
$$21 = 14 + x \Rightarrow x = 7$$

Thus $AC = AR + RC = 6 + 7 = 13$ cm

$BC = BQ + QC = 8 + 7 = 15$ cm

OR

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Ans :

A circle centre O is inscribed in a quadrilateral $ABCD$ as shown in figure given below.

Since OE and OF are radius of circle,

$$OE = OF$$

Tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

Thus $\angle OEA = \angle OFA = 90^\circ$

Now in ΔAEO and ΔAFO,

$OE = OF$

$\angle OEA = \angle OFA = 90^\circ$

$OA = OA$ (Common side)

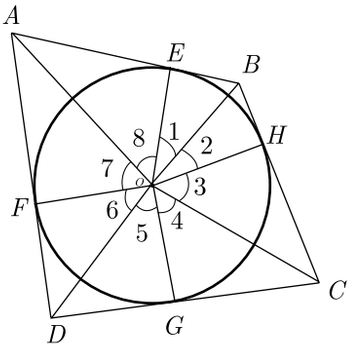
Thus $\Delta AEO \cong \Delta AFO$ (SAS congruency)

$\angle 7 = \angle 8$

Similarly, $\angle 1 = \angle 2$

$\angle 3 = \angle 4$

$\angle 5 = \angle 6$



Since angle around a point is 360°,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$

$2\angle 1 + 2\angle 8 + 2\angle 4 + 2\angle 5 = 360^\circ$

$\angle 1 + \angle 8 + \angle 4 + \angle 5 = 180^\circ$

$(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180^\circ$

$\angle AOB + \angle COD = 180^\circ$ Hence Proved.

13. The advantages of cone bottom tanks are found in nearly every industry, especially where getting every last drop from the tank is important. This type of tank has excellent geometry for draining, especially with high solids content slurries as these cone tanks provide a better full-drain solution. The conical tank eliminates many of the problems that flat base tanks have as the base of the tank is sloped towards the centre giving the greatest possible full-drain system in vertical tank design.

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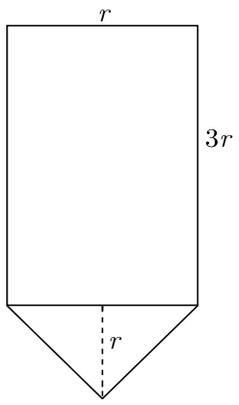
Rajesh has been given the task of designing a conical bottom tank for his client. Height of conical part is equal to its radius. Length of cylindrical part is the 3 times of its radius. Tank is closed from top. The cross section of conical tank is given below.



(i) If radius of cylindrical part is taken as 3 meter, what is the volume of above conical tank ?

(ii) What is the area of metal sheet used to make this conical tank ? Assume that tank is covered from top.

Ans :

(i) Length of cylindrical part is three times of radius of conical part and height of conical part is equal to its radius.
If we assume r be the common radius of cylindrical part and conical part, height of conical part will be r and length of cylindrical part will be $3r$.

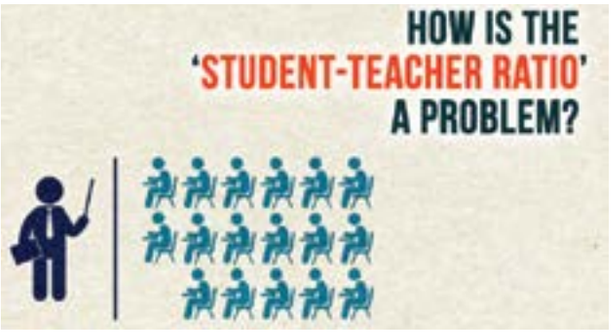
Volume of conical tank= Volume of cylindrical part + Volume of conical part

$$= \pi r^2 l + \frac{1}{3}\pi r^2 h$$
$$= \pi r^2 \cdot 3r + \frac{1}{3}\pi r^2 \cdot r$$
$$= 3\pi r^3 + \frac{1}{3}\pi r^3 = \frac{10}{3}\pi r^3$$
$$= \frac{10}{3}\pi (3)^3 = 90\,\pi \text{ m}^3$$

(ii) Surface area of tank = SA of top + CSA of cylinder + CSA of cone

$$= \pi r^2 + 2\pi rl + \pi r\sqrt{h^2 + r^2}$$
$$= \pi r^2 + 2\pi r \cdot 3r + \pi r\sqrt{r^2 + r^2}$$
$$= \pi r^2 + 6\pi r^2 + \sqrt{2}\,\pi r^2$$
$$= (1 + 6 + \sqrt{2})\,\pi r^2$$
$$= (7 + \sqrt{2})\,\pi (3)^2$$
$$= 9(7 + \sqrt{2})\,\pi \text{ m}^2$$

14. Student-teacher ratio expresses the relationship between the number of students enrolled in a school and the number teachers employed by the school. Student-teacher ratio is important for a number of reasons. It can be used as a tool to measure teacher workload as well as the allocation of resources. A low student-teacher ratio indicates the burden on a single teacher of teaching multiple students as well as the lack of time that each student gets.



A survey was conducted in the 100 secondary school of Rajasthan and following frequency distribution table was prepared

Students per teacher	Number of School
20-25	5
25-30	15
30-35	25
35-40	30
40-45	15
45-50	10

(i) What is the median value of students per teacher?

(ii) What is the model value of students per teacher ?

Ans :

(i) Median, $M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$

$$= 35 + \frac{50 - 45}{30} \times 5$$
$$= 35 + \frac{5}{6} = \frac{215}{6} = 35.83$$

(ii) Here, $l = 35$, $f_1 = 30$, $f_0 = 25$, $f_2 = 15$ and $h = 5$

Mode, $M_o = l + h\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$

$$= 35 + \frac{30 - 25}{60 - 25 - 15} \times 5$$
$$= 35 + \frac{5}{20} \times 5$$
$$= 35 + 1.25 = 36.25$$

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